Exponential and Logistic Functions

Definition:

Let a and b be real number constants. An exponential function in x is a function that can be written in the form, $f(x) = a \cdot b^x$ where a is non zero, b is positive, and $b \neq 1$. The constant a is the initial value of f (the value at $x = 0$), and b is the base.

EX#1: Which of the following are exponential functions? Why?

Identifying Exponential Functions			
Function	**Initial Value**	**Base**	**Why?**
$f(x) = 2^x$	1	2	yes, in $y = ab^x$ form
$g(x) = 4x^{-3}$	4	x	no, rational function $y = \frac{a}{x^n}$
$h(x) = 5(2^{-x})$	5	2	yes, in $y = ab^x$ form
$f(x) = 3 \cdot 4^e$	3	4	no, this is a constant function

EX #2: Compute the exact value without using a calculator.

$f(x) = -5(8)^x$ for $x = \frac{1}{3}$

$$f(x) = -5(8)^{1/3}$$
$$f(\tfrac{1}{3}) = -5\sqrt[3]{8}$$
$$f(\tfrac{1}{3}) = -5(2)$$
$$f(\tfrac{1}{3}) = -10$$

Finding an Exponential Function from its Table of Values

EX #3: Determine the formulas for the functions from values given in the table below.

Values for Two Exponential Functions		
x	$f(x)$	$g(x)$
-2	$\frac{3}{4}$ $\rangle \cdot 2$	20 $\rangle \div 2$
-1	$\frac{3}{2}$ $\rangle \cdot 2$	10 $\rangle \div 2$
→ 0	3 $\rangle \cdot 2$	5 $\rangle \div 2$
1	6 $\rangle \cdot 2$	$\frac{5}{2}$ $\rangle \div 2$
2	12	$\frac{5}{4}$

model
$f(x) = a \cdot b^x$
a = initial value
$(0, a)$
$b^x \Rightarrow$ base (pattern)

$f(x)$: growth
$a = 3$
b = multiply by 2
$\boxed{f(x) = 3 \cdot 2^x}$

$g(x)$: decay
$a = 5$
b = divide by 2
$g(x) = 5\left(\frac{1}{2}\right)^x$

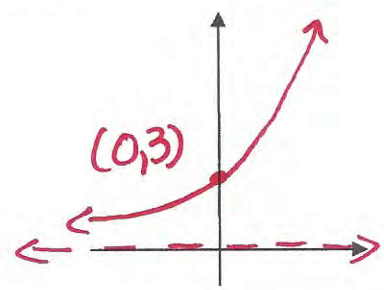
Exponential Growth and Decay

For any exponential function $f(x) = a \cdot b^x$ and any real number x, $f(x+1) = b \cdot f(x)$.

If $a > 0$ and $b > 1$, the function f is increasing and is an **exponential** growth **function.** The base b is its growth **factor.**

EX: $f(x) = 3 \cdot 2^x$

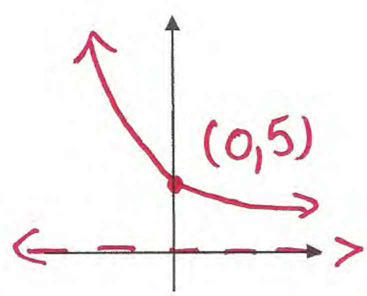
H.A. $y = 0$; domain $(-\infty, \infty)$
range $(0, \infty)$



If $a > 0$ and $b < 1$, the function f is decreasing and is an **exponential** decay **function.** The base b is its decay **factor.**

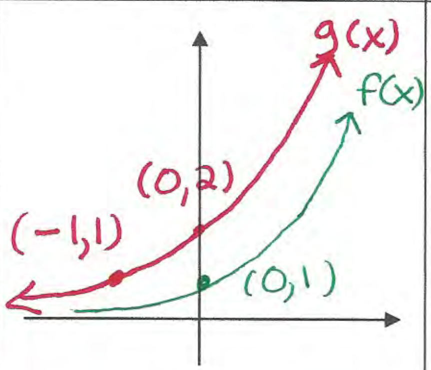
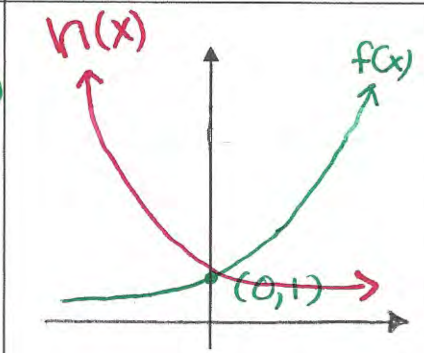
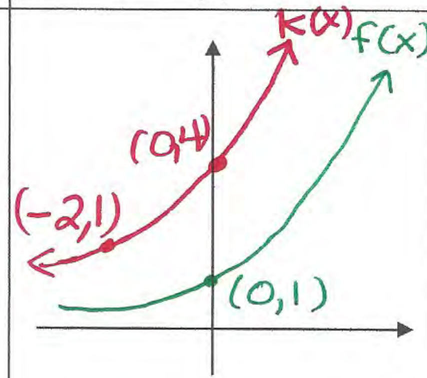
EX: $g(x) = 5\left(\frac{1}{2}\right)^x$

H.A. $y = 0$; domain $(-\infty, \infty)$
range $(0, \infty)$



Transforming Exponential Functions

EX #4: Describe how to transform the graph of $f(x) = 2^x$ into the graph of the given functions.

A. $g(x) = 2^{x+1}$	**B.** $h(x) = 2^{-x}$	**C.** $k(x) = 4 \cdot 2^x$
		
(0,1) moves 1 unit left g(0) = 2	(0,1) does not move, h(x) is reflected @ y-axis	(0,1) moves 2 units left k(0) = 4

The Natural Base e

e is the 2nd most famous irrational number is the sum of the series: $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$

$$\sum_{k=0}^{\infty} \frac{1}{k!} = e$$

DEFINITION: $\boxed{e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x}$

Any exponential function $f(x) = a \cdot b^x$ can be rewritten as $f(x) = a \cdot e^{kx}$, for an appropriately chosen real constant k. This form is an

exponential growth function: If $a > 0$ and $k > 0$

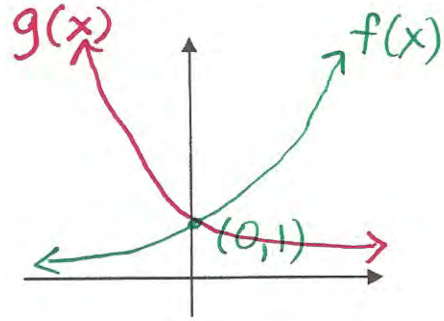
exponential decay function: If $a > 0$ and $k < 0$

$\boxed{\text{Ln means NATURAL LOG and uses BASE } e}$

EX #5: Describe how to transform the graph of $f(x)=e^x$ into the graph of the given function.

A. $g(x)=e^{-2x}$

faster decay + reflected across the y-axis



* $e \approx 2.71828\ldots$
* $\pi \approx 3.14159\ldots$

} constants not variables

Logistic Growth Functions

Let *a, b, c,* and *k* be positive constants, with $b<1$. A logistic **function** in *x* is a function that can be written in the form:

$$\boxed{f(x)=\frac{c}{1+a\cdot b^x}} \quad \text{or} \quad \boxed{f(x)=\frac{c}{1+a\cdot e^{-kx}}}$$

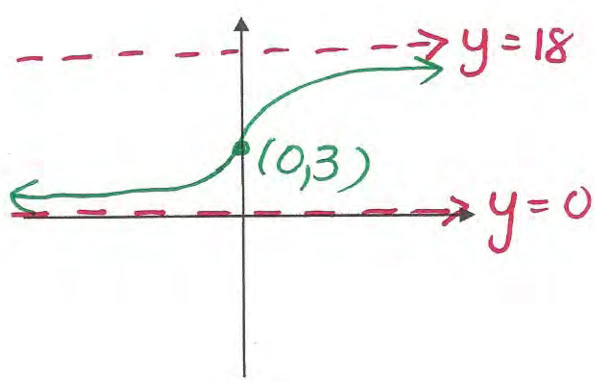
where the constant *c* is the limit to growth or carrying capacity.

EX #6: Graph the function. Find the *y*-intercept and the horizontal asymptotes.

$$f(x)=\frac{18}{1+5\cdot 0.2^x}.$$

y-intercept: $f(0)=\frac{18}{1+5(.2)^0}$

$f(0)=3$



Horizontal Asymptotes:

$y=18$

$y=0$

A Flu Epidemic Model

EX #7: The function $f(t) = \dfrac{64{,}000}{1+4500e^{-1.2t}}$ describes the number of people, $f(t)$, who have become ill with a flu outbreak t weeks after its initial discovery within a town of 64,000 people.

A. How many people were ill at the beginning of the epidemic?

y-intercept

$$f(0) = \frac{64{,}000}{1+4500e^{0}}$$

$$\boxed{f(0) \approx 14 \text{ people}}$$

$$f(0) = \frac{64{,}000}{4501}$$

B. How many people were ill by the end of the fourth week?

$f(4) \approx 1682$ people

$$f(4) \approx \frac{64{,}000}{1+4500e^{-1.2(4)}}$$

C. What is the limiting size of $f(t)$, the population that can become ill?

64,000 people is the limiting factor.