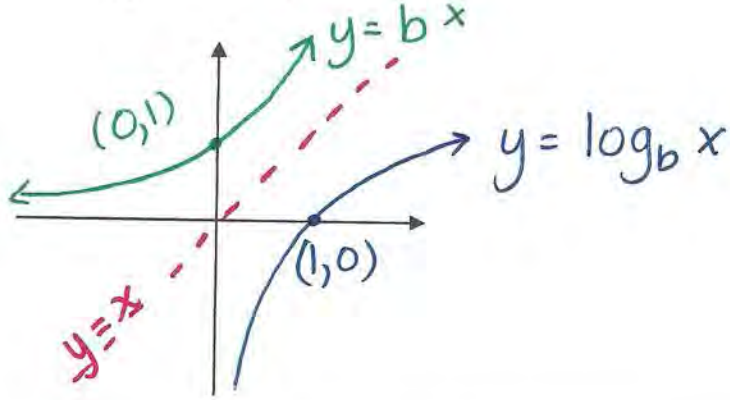
Logarithmic Functions and Their Graphs

Logarithmic Functions are Inverses of Exponential Functions

If $a > 0$ and $b > 0$, $b \neq 1$, then $y = log_b x$ if and only if $b^y = x$.

Graph: $y = b^x$
$y = log_b x$
$y = x$



Basic Properties of Logarithms (Note: These are also valid for base 10 and base e) For $b > 0, b \neq 1$, $x > 0$ and any real number y			
1.	$log_b 1 = 0$	because . . .	$b^0 = 1$
2.	$log_b b = 1$	because . . .	$b^1 = b$
3.	$log_b b^y = y$	because . . .	$b^y = b^y$
4.	$b^{log_b x} = x$	because . . .	$log_b x = log_b x$

Evaluating Logarithmic and Exponential Expressions

EX #1: Evaluate the following logarithms			
A	$log_2 8 = 3$	because . . .	$2^3 = 8$
B	$log_3 \sqrt{3} = \frac{1}{2}$	because . . .	$3^{1/2} = \sqrt{3}$
C	$log_5 \frac{1}{25} = -2$	because . . .	$5^{-2} = \frac{1}{25}$
D	$log_7 7 = 1$	because . . .	$7^1 = 7$
E	$6^{log_6 11} = 11$	because . . .	$log_6 11 = log_6 11$

Logarithmic functions are inverses of exponential functions (x & y are switched)

x	$f(x) = 2^x$	x	$f^{-1}(x) = \log_2 x$
−2	$\frac{1}{4}$	$\frac{1}{4}$	−2
−1	$\frac{1}{2}$	$\frac{1}{2}$	−1
0	1	1	0
1	2	2	1
2	4	4	2

Common Logarithms, Base 10

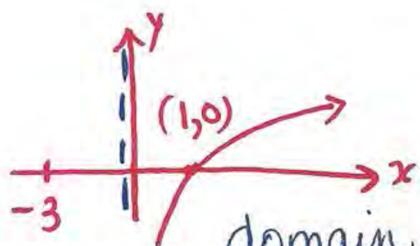
Logarithms with base 10 are called common logarithms. The subscript 10 is often dropped, so a logarithmic statement with no specified base is understood to be base 10.

EX #2: Evaluate the following logarithms and exponential expressions.

A. $\log 100$ 2 $10^x = 10^2$	B. $\log\sqrt{10}$ $\frac{1}{2}$ $10^x = 10^{1/2}$
C. $\log \frac{1}{1000}$ −3 $10^x = 10^{-3}$	D. $10^{\log_6 6}$ 1 $6^1 = 6^1$

Ex #3: Evaluate these common logarithms with a calculator.

A. $\log 34.5$ 1.538	**B.** $\log 0.43$ −0.367	**C.** $\log(-3)$ not defined



domain $(0,\infty)$

Solving Simple Logarithmic Equations

To solve an exponential equation, change it to a logarithmic equation. To solve a logarithmic equation, change it to an exponential equation.

Ex #4: Solve each equation by changing it to exponential form.

A. $\log x = 3$ $10^3 = x$ $x = 1000$	**B.** $\log_2 x = 5$ $2^5 = x$ $x = 32$

Natural Logarithms, Base e

Notation: The logarithmic function $\log_e x = \ln x$.

property
$e^{\ln N} = N$

EX #5: Evaluate the following logarithmic and exponential expressions.

A. $\ln\sqrt{e}$ $e^x = e^{1/2}$ $x = 1/2$	**B.** $\ln e^5$ $e^x = e^5$ $x = 5$	**C.** $e^{\ln 4}$ $\ln x = \ln 4$ $x = 4$

Evaluating Natural Logarithms with a Calculator

EX #6: Use a calculator to evaluate the logarithmic expressions.

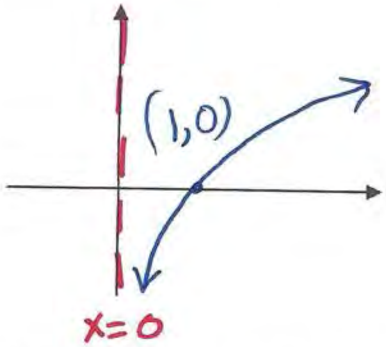
A. $\ln 23.5$ 3.157	**B.** $\ln 0.48$ −0.734	**C.** $\ln(-5)$ not defined

Basic Properties for Natural Logarithms			
1.	$\ln 1 = 0$	because . . .	$e^0 = 1$
2.	$\ln e = 1$	because . . .	$e^1 = e$
3.	$\ln e^y = y$	because . . .	$e^y = e^y$
4.	$e^{\ln x} = x$	because . . .	$\ln x = \ln x$

Graphing Logarithmic Functions

Basic Function: $f(x) = \ln x$

Graph:



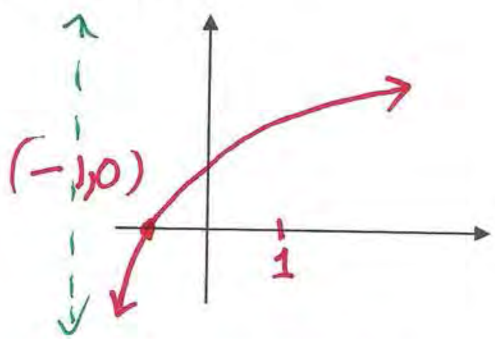
Analysis:

domain $(0, \infty)$
range $(-\infty, \infty)$
continuous, increasing
no symmetry, no extrema
unbounded, vertical asymptote $x=0$
$\lim_{x \to \infty} f(x) = \infty$

EX #7: Transforming Logarithmic Graphs

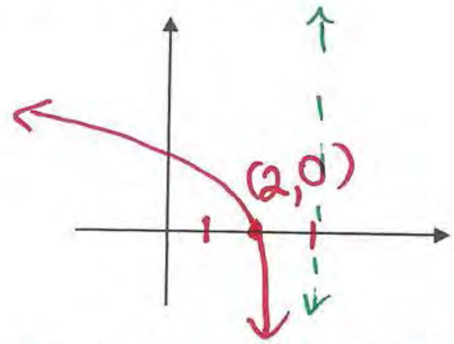
Describe how to transform the graph of $f(x) = \ln x$ or $y = \log x$ into the graph of the given function.

A. $g(x) = \ln(x+2)$



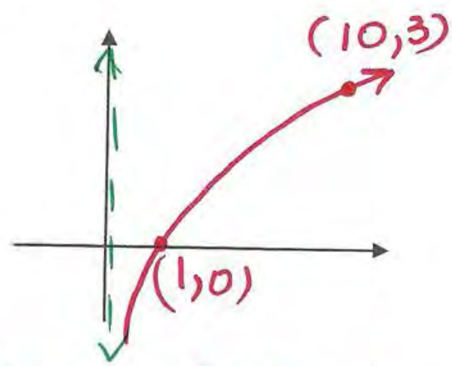
left 2 units
$x = -2$ V.A.

B. $h(x) = \ln(3-x)$



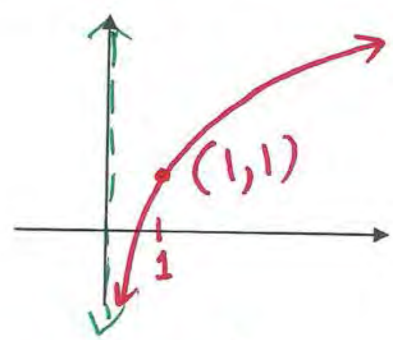
$\ln(-(x-3))$ reflect over y axis, then 3 units right, $x=3$ V.A

C. $g(x) = 3\log x$



stretch factor 3
$X=0$ V.A.

D. $h(x) = 1 + \log x$



up 1 unit.
$X=0$ V.A.